ROBUST CONTROL OF HYSTERETIC BASE-ISOLATED STRUCTURES UNDER SEISMIC DISTURBANCES

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The System to be Controlled
Previous Work

- It can be seen in the references* that there are controllers available for this application.

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  $$\max |u(t)| \leq 2m$$

  $$\forall t \geq 0$$
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$$\forall t \geq 0$$

System Model

\[ m\ddot{x} + c\dot{x} + \Phi(x, t) = f(t) + u(t) \]
\[ \Phi(x, t) = \alpha Kx(t) + (1 - \alpha) DKz(t) \]
\[ \dot{z} = D^{-1} \left( A\dot{x} - \beta |\dot{x}| \dot{z}^{n-1} z - \gamma \dot{x}|z|^n \right) \]
System Model

\[
\begin{align*}
    m\ddot{x} + c\dot{x} + \Phi(x, t) &= f(t) + u(t) \\
    \Phi(x, t) &= \alpha K x(t) + (1 - \alpha) D K z(t) \\
    \dot{z} &= D^{-1} (A \dot{x} - \beta |\dot{x}| |\dot{z}|^{n-1} z - \gamma |x| z^n)
\end{align*}
\]

\[K > 0 \quad n \geq 1\]
\[D > 0 \quad \alpha \in (0, 1)\]

The parameters \(A, \beta\) and \(\gamma\) determine the hysteretic behavior of the system.*

System Assumptions

- The disturbance is unknown, but bounded:

\[ |f(t)| \leq F \]
\[ \forall t \geq 0 \]
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  \[ |f(t)| \leq F \]
  \[ \forall t \geq 0 \]

- The structure is at rest before the seism occurs:
  \[ z(0) = x(0) = \dot{x}(0) = 0 \]
The displacement and the inner dynamic of the open loop system are bounded:

\[
|x(t)| \leq \bar{\rho}_x \\
\forall t \geq 0
\]

\[
|z(t)| \leq \bar{\rho}_z \\
\forall t \geq 0
\]
System Assumptions

- The displacement and the inner dynamic of the open loop system are bounded:

\[ |x(t)| \leq \bar{\rho}_x \quad \forall t \geq 0 \]
\[ |z(t)| \leq \bar{\rho}_z \quad \forall t \geq 0 \]

- The displacement and velocity are available for use in the control law:

\[ x, \dot{x} \rightarrow u \]
Control Objective

To find a suitable control law such that the velocity and displacement, are ultimately within a given bound:
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To find a suitable control law such that the velocity and displacement are ultimately within a given bound:

$$\lim_{t \to \infty} (|x(t)|^2 + |\dot{x}(t)|^2) \leq \bar{a}$$
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To find a suitable control law such that the velocity and displacement, are ultimately within a given bound:

$$\lim_{t \to \infty} (|x(t)|^2 + |\dot{x}(t)|^2) \leq \bar{a}$$

where $\bar{a}$ is a positive constant
Proposed Control Law

Theorem: The control law:

\[ u = -k_1x - \varrho \frac{(\dot{x} + \epsilon x)}{\dot{x} + \epsilon x} + \tau \]
Proposed Control Law

**Theorem:** The control law:

\[
u = -k_1 x - \rho \frac{(\dot{x} + \epsilon x)}{\vert \dot{x} + \epsilon x \vert + \tau}\]

given the conditions:

\[
k_1 > 0 \quad \epsilon > 0
\]
\[
\epsilon < \sqrt{\frac{k_1}{m}} \quad k_1 > \frac{c^2}{4(c - \epsilon m)}
\]
\[
\vert \Phi(x, t) \vert + \vert f(t) \vert < \rho
\]
Proposed Control Law (contd.)

achieves the control objective with:

\[ \bar{a} = \varrho \tau \lambda_{\text{max}} \left( A_1 \right) \lambda_{\text{min}} \left( A_2 \right) \]

\[ A_1 = \left( k_1 \epsilon \right), \quad A_2 = \begin{bmatrix} k_1 \epsilon_1 \\ c \epsilon_1 \end{bmatrix} \left( c - \epsilon_m \right) \]
Proposed Control Law (contd.)

achieves the control objective with:

\[ \bar{a} = \frac{\varrho \tau \lambda_{\text{max}}(A_1)}{\lambda_{\text{min}}(A_1) \lambda_{\text{min}}(A_2)} \]
Proposed Control Law (contd.)

achieves the control objective with:

\[ \bar{a} = \frac{\varrho \tau \lambda_{\text{max}}(A_1)}{\lambda_{\text{min}}(A_1) \lambda_{\text{min}}(A_2)} \]

\[ A_1 = \begin{pmatrix} k_1 & \epsilon m \\ \epsilon m & m \end{pmatrix}, \quad A_2 = \begin{pmatrix} k_1 \epsilon & 1 \\ \frac{1}{2} c \epsilon & (c - \epsilon m) \end{pmatrix} \]
System Parameters

The following parameters were used to simulate the system*:

\[ A = 1 \quad \beta = 0.1 \text{m/s} \]
\[ \gamma = 0.5 \quad n = 3 \]
\[ \alpha = 0.6 \quad m = 156 \times 10^3 \text{ kg} \]
\[ D = 0.6 \quad K = 10^6 \text{N/m} \]
\[ c = 2 \times 10^4 \text{Ns/m} \]

The 1940 El Centro, CA, earthquake was used in the simulations to test the controller.
Response of the Open-Loop System
Controller Parameters

The following parameters were used to simulate the active controller*:

\[
\bar{\rho}_x = 0.03 \text{m} \quad \bar{\rho}_z = 1.19 \text{m/s} \\
\tau = 0.01 \quad k_1 = 3 \times 10^4 \\
F = 0.3 \quad \epsilon = 0.1 \\
\varrho = 2 \times 10^7
\]

Response of the Controlled System

\[ \max |u(t)| \leq \frac{m}{2} \]
\[ \forall t \geq 0 \]
Controller Adjustment

The system model can be rewritten as:

\[

m\ddot{x} + c\dot{x} + \alpha Kx = u(t) + \delta_1(x, t)
\]

\[

\delta_1(x, t) = f(t) - (1 - \alpha)DKz(t)
\]
Controller Adjustment

The system model can be rewritten as:

\[ m\ddot{x} + c\dot{x} + \alpha Kx = u(t) + \delta_1(x, t) \]

\[ \delta_1(x, t) = f(t) - (1 - \alpha)DKz(t) \]

So we have:

\[ |\delta_1(x, t)| \leq |\delta(x, t)| \leq \varrho \]
Controller Adjustment

The system model can be rewritten as:

\[ m\ddot{x} + c\dot{x} + \alpha Kx = u(t) + \delta_1(x, t) \]

\[ \delta_1(x, t) = f(t) - (1 - \alpha)DKz(t) \]

So we have:

\[ |\delta_1(x, t)| \leq |\delta(x, t)| \leq \varrho \]

Under certain conditions for \( \alpha K \), this product can replace \( k_1 \), resulting in:

\[ u = -\varrho \frac{(\dot{x} + \epsilon x)}{(|\dot{x} + \epsilon x| + \tau)} \]
Response of the System to the Adjusted Controller
Further Considerations

In an attempt to reduce the amount of energy required by the controller, the term “$\varrho$”, which is a conservative bound, was reduced, achieving a reduction in the energy required by the controller, but this also resulted in an increased displacement, as shown in the following slides.
Response of the System to the Adjusted Controller

\[ \varrho = 2 \times 10^5 \quad \text{max} \left| u(t) \right| \approx \frac{m}{3} \quad \forall t \geq 0 \]
Response of the System to the Adjusted Controller

\[ \varphi = 2 \times 10^4 \]

\[ \max |u(t)| \leq \frac{m}{10} \quad \forall t \geq 0 \]
Conclusions

- The Control Objective is achieved with a simple controller.
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- It is possible to implement a strategy to reduce energy requirements.
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- The Control Objective is achieved with a simple controller.
- It is possible to implement a strategy to reduce energy requirements.
- Knowledge of upper bounds is required.
Questions?

Thank you for your attention